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Two-dimensional heat transfer analysis of radiating plates

K. BADARI NARAYANA and S. U. KUMARI
Thermal Systems Division, ISRO Satellite Centre, Vimanapura Post, Bangalore 560 017, India

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Abstract—The strongly implicit procedure is used to solve non-linear elliptical two-dimensional heat conduction in radiating plates. A uniform heat flux is applied at one end of the plate which dissipates heat by radiation from one end into a vacuum at 0 K and into an ambient at temperature T_i from the other. The results are presented with reference to four non-dimensional parameters.

1. INTRODUCTION

EFFICIENT heat removal systems are required for the safe and satisfactory operation of spacecraft. In certain cases the surface area of the heat dissipating electronic equipment may not be sufficient to transfer the heat. In such circumstances they are mounted on high conductivity plates to enhance the heat transfer. Cooling devices normally termed as radiators, doublers are used. The accurate prediction of the thermal performance of these is essential for compact and efficient design. However, the analysis of such contrivances is conventionally based upon several simplifying assumptions, in particular, that the heat flow is unidirectional. Many investigators have reported the one-dimensional heat flow analyses of radiators [1–5]. In practice the configuration of the plate used and the heat footprint location make a two-dimensional heat flow study mandatory. Such studies, however, are very scarce in the literature.

A two-dimensional study of fin tube radiators is reported in refs. [6, 7]. Sikka and Iqbal [8] have given a series solution the coefficients of which are determined by the least squares fit method, to analyse the two-dimensional heat flow in a circular radiating fin. Both convective and radiative heat dissipations without incident radiation were considered. A variational formulation for analysing the two-dimensional temperature distribution in a rectangular solid receiving radiant heat flux on one face and emanating radiant energy to the atmosphere at 0 K is given by Iqbal and Aggarwala [9]. Moszynski and Champaneria [10] have given a semi-iterative solution procedure for the determination of the two-dimensional temperature distribution in radiating fins. No generation of heat within is considered in the above analyses. More recently Bobco and Starkovs [11] have developed closed form solutions to radiating plates comparable to thermal doublers. They simplified the problem by linearizing the radiation term. Such an approximation is applicable only where the maximum doubler temperature is not significantly higher than the immediate sink temperature.

In summary, the study of two-dimensional radiating plates is scarce. The few results which are available in the open literature are far from complete in as much as the methods still involve fairly bold assumptions. In this paper, numerical solutions are developed for two-dimensional radiating plates receiving a uniform heat flux at one surface and sun load at the other and radiating both to ambient and vacuum space conditions simultaneously.

2. FORMULATION

A schematic representation of the system considered is shown in Fig. 1. The theoretical representation of this is developed on the basis of the following assumptions:

(1) the heat flow is steady;
(2) the plate material is isotropic;
(3) the plate radiates from one end to a vacuum space which is at a constant arbitrary temperature of 0 K, and to the ambient at a uniform temperature T_i from the other;
(4) all radiating surfaces are grey and diffuse;
(5) the heat flux applied over any footprint area is uniform and the contact resistance between the heat application zone and the plate is negligible;
(6) the plate receives sun load on the surface facing the vacuum space
(7) the thickness of the plate is very small compared to other dimensions.

For two-dimensional heat conduction the energy and boundary condition equations are as follows.

Heat application zone

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{K_3}{k\delta}+\frac{q}{k\delta}-\frac{K_2 T^4}{k\delta}=0. \qquad (1a)$$

Remaining zone

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{K_3}{k\delta}-\frac{K_1}{k\delta}(T^4-T_i^4)-\frac{K_2 T^4}{k\delta}=0 \quad (1b)$$

NOMENCLATURE

- A matrix coefficient
- $\tilde{A}$ auxiliary matrix
- $a_{ij}, \ldots, e_{ij}$ coefficients of difference equation
- B dimensionless breadth, b/x_o
- b breadth of the plate [m]
- I solar constant [W m^{-2}]
- K_1, K_2, K_3 coefficients, equation (1)
- k thermal conductivity [W m^{-1} K^{-1}]
- L dimensionless length, l/x_o
- l length of the plate [m]
- q heat flux [W m^{-2}]
- S source vector, equation (7)
- $s_{i,j}$ source term
- T temperature [K]
- T_o reference temperature [K]
- U dimensionless temperature, T/T_o
- W emissivity ratio, $\varepsilon_i/\varepsilon_o$
- X, Y dimensionless Cartesian coordinates, equation (5)
- x, y Cartesian coordinates [m]
- x_o reference length [m].

Greek symbols

- α solar absorptivity
- δ thickness of the plate [m]
- ε infra-red emissivity
- θ solar angle
- ξ dimensionless profile factor, $K_2 x_o^2 T_o^3/k\delta$
- σ Stefan–Boltzmann constant [W m^{-2} K^{-4}]
- ϕ dimensionless heat flux, $qx_o^2/k\delta T_o$
- ψ dimensionless environment factor, $K_3/K_2 T_o^4$
- ω_n relaxation parameter, equation (11)
- ω_s relaxation parameter, equation (9).

Superscript

- * guess value.

Subscripts

- i inside
- o outside
- p present.

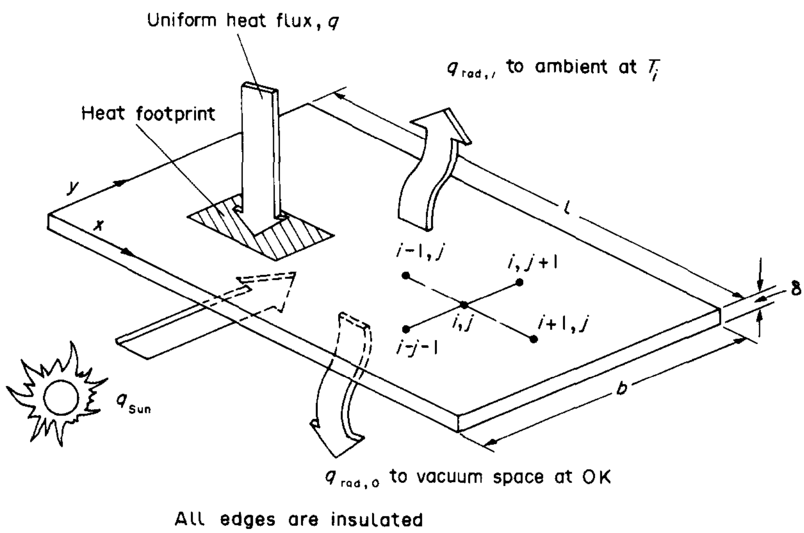

FIG. 1. Schematic diagram of the radiating plate.

where

$$K_1 = \sigma\varepsilon_i, K_2 = \sigma\varepsilon_o \text{ and } K_3 = I \propto \sin\theta.$$

Boundary conditions

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x = 0 \text{ and } l \tag{2a}$$

$$\frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0 \text{ and } b. \tag{2b}$$

The non-linear elliptic problem defined by equations (1) and (2), is now given in dimensionless form.

Heat application zone

$$\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \phi - \xi(U^4 - \psi) = 0. \tag{3a}$$

Remaining zone

$$\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} - \xi[(1+W)U^4 - WU_i^4 - \psi] = 0. \tag{3b}$$

Boundary conditions

$$\frac{\partial U}{\partial X} = 0 \quad \text{at } X = 0 \text{ and } L \tag{4a}$$

$$\frac{\partial U}{\partial Y} = 0 \quad \text{at } Y = 0 \text{ and } B \tag{4b}$$

where

$$U = \frac{T}{T_o}, \quad X = \frac{x}{x_o}, \quad Y = \frac{y}{x_o}$$

$$\phi = \frac{qx_o^2}{k\delta T_o}, \quad \xi = \frac{K_2 x_o^2 T_o^3}{k\delta}, \quad \psi = \frac{K_3}{K_2 T_o^4} \tag{5}$$

and

$$W = \frac{K_1}{K_2}.$$

3. NUMERICAL SOLUTION

The solution to the problem described by equations (3) and (4) is susceptible to treatment by various numerical techniques. The boundary integral equation and series truncation method is used by some investigators [12]. An implicit finite difference scheme is used in the present investigation to write equation (3) in the well-known five-point difference equation as

$$a_{ij} U_{i,j-1} + b_{ij} U_{i-1,j} + c_{ij} U_{i,j} + d_{ij} U_{i+1,j} + e_{ij} U_{i,j+1} = s_{i,j}. \tag{6}$$

The difference equation (6) is expressed in matrix form as

$$AU = S \tag{7}$$

where A represents a_{ij} to e_{ij}, U, all U_{ij}'s and S, all s_{ij}'s.

Due to the limitations of the normally used Jacobi iterative, Gauss–Seidel iterative, successive over relaxation and the ADI methods in solving the large nonlinear set of equations obtained here the strongly implicit procedure of Stone [13] is used in the present investigation. The principal advantage is the faster convergence rate of this compared to the other previously mentioned methods.

3.1. *The strongly implicit procedure (SIP)*

A general iterative formula for equation (6) is obtained by adding an auxiliary term $\tilde{A}$ to each side of equation (7) and setting the iteration number to U as

$$[A+\tilde{A}]\{U\}^{n+1} = [A]\{U\}^n + \{S\} \tag{8}$$

where n is the number of iterations and the form of $[\tilde{A}]$ is such that $\|[\tilde{A}]\| \ll \|[A]\|$ and the decomposition of $[A+\tilde{A}]$ into a lower and an upper triangular matrix product involves much less computation than the direct decomposition of $[A]$. Factorization of $[A+\tilde{A}]$ into $[L]+[U]$ or $[L]+[B]$, where $[L]$, $[U]$ and $[B]$ are respectively lower, upper and diagonal matrix, reduces the method to point Jacobi iterative and Gauss–Seidel schemes, respectively. In the SIP method however, $[A+\tilde{A}]$ is factorized as $[L]\cdot[U]$. Since the right-hand side of equation (8) involves the unknown solution vector $\{U\}$, the following iteration scheme of Stone [13] is used:

$$[A+\tilde{A}]\{U\}^{n+1} = [A+\tilde{A}]\{U\}^n - \omega_s([A]\{U\}^n - \{S\}) \tag{9}$$

where ω_s is the relaxation parameter.

3.2. *Source term linearization*

To avoid the steady drift or oscillation with increasing amplitude of the computed 'U' values the source term is linearized by splitting S as follows [14]:

$$s = s^* + \left(\frac{ds}{dU}\right)^* (U_p - U_p^*) \tag{10}$$

where the superscript denotes the guess or previous iteration value. Also to accelerate the convergence rate a relaxation parameter is used for the source and the coefficient c_{ij} during the recalculation. Source term relaxation for example is written as

$$s_{ij} = \omega_n s_{ij} + (1-\omega_n) s_{ij}^*. \tag{11}$$

The convergence criterion set for the U value is less than or equal to 10^{-8}. The linearized source equations are solved repeatedly in an outer iterative cycle and the SIP method is evaluated in an inner cycle.

3.3. *Grid generation and treating irregular boundaries*

The given irregular shaped plate is approximated to a polygon and is considered in a positive coordinate system. A suitable rectangular grid is then superimposed over the polygon. By making use of the direction cosines of the directed line segments from the grid points they are designated as external, internal and boundary nodes. Depending on the type of nodes they are represented by a number. Based on the above procedure the number of grid points of the five point molecule difference representation which are outside the domain under consideration is determined. The method proposed by Fox as in ref. [15] is used to handle the derivative normal to the irregular boundaries. A computer program is developed for the automatic generation of the grid and the details are presented elsewhere [16].

4. RESULTS AND DISCUSSION

Two-dimensional temperature distribution in radiating plates is computed for different values of the relaxation parameters ω_n and ω_s. The optimum values are 0.95 and 0.1, respectively. The consistency of the computer program developed is confirmed by checking the independency of the converged solution for different values of the initial guesses of temperature,

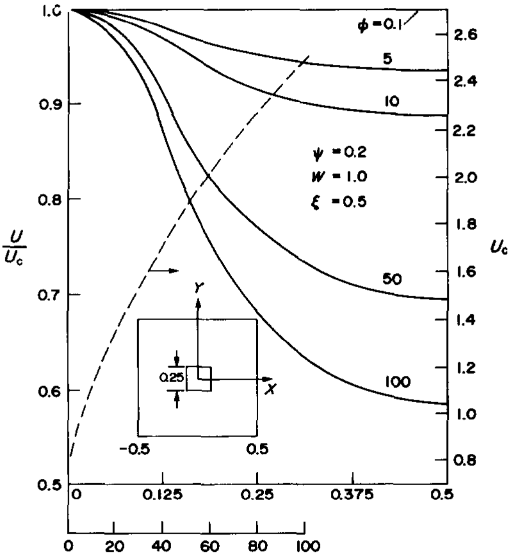

FIG. 2. Effects of heat flux parameter ϕ on temperature.

the relaxation factors, the reference temperature and coordinate axes values. The deviation in temperature value at the centre and the corner of a square plate radiating to space is found to be less than 0.25% when the grid size is quadrupled from 8×8 to 32×32. In most of the computations however, the minimum number of divisions considered along any of the axes is 16.

Figure 2 shows the effect of the normalized heat flux parameter ϕ on the variation of temperature with distance, where U_c corresponds to the normalized centre temperature of the square radiating plate considered. The range of ϕ considered is 0.1–100. The

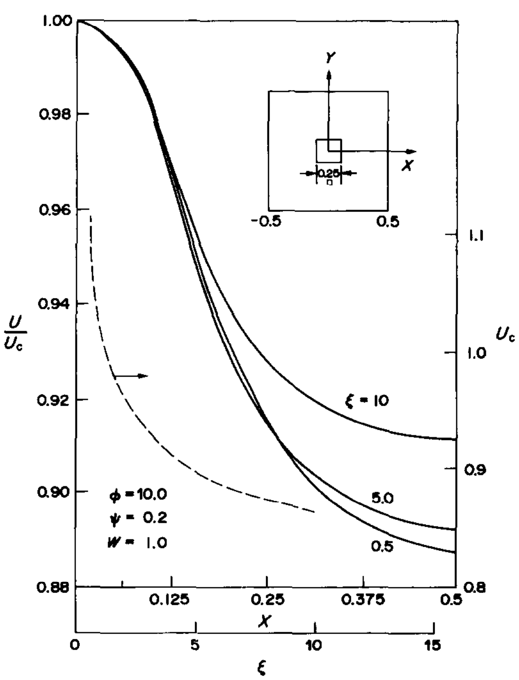

FIG. 3. Effects of emissivity factor ξ on temperature.

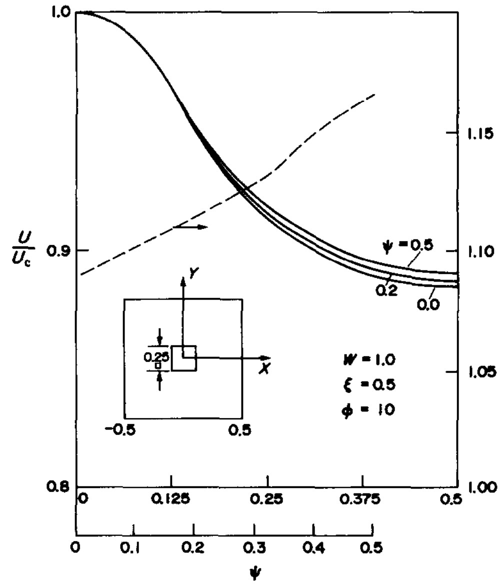

FIG. 4. Effects of environment parameter ψ on temperature.

temperature gradient and centre temperature increases with heat flux. Though the term $k\delta$ appears both in ϕ and ξ, the figure shows the possible trend in the temperature distribution for changes in the material property. The term ξ is sometimes considered as the radiation Biot number. However, because of the appearance of terms k, δ and T_0 in both ϕ and ξ, it can be treated as an emissivity factor of the radiating surface. The temperature profile flattens and the origin temperature reduces with an increase in ξ (Fig. 3).

The environment parameter ψ is the normalized sun load falling on the plate. At $\psi = 0$ no sun load falls on the plate. The overall effect of the variation of ψ on normalized temperature is small as shown in Fig. 4. The radiating plate is interacting with vacuum space and ambient conditions on either sides excepting the heat footprint zone, where a uniform heat flux is applied. For a given area of the plate and heat load, the average temperature level of the plate can be reduced by effectively radiating from either sides of the plate. A measure of this is expressed as the emis-

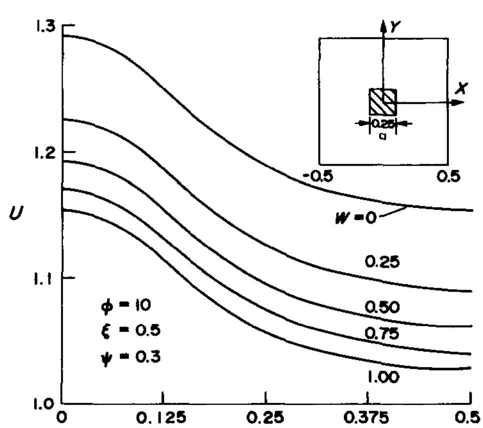

FIG. 5. Effects of emissivity ratio W on temperature.

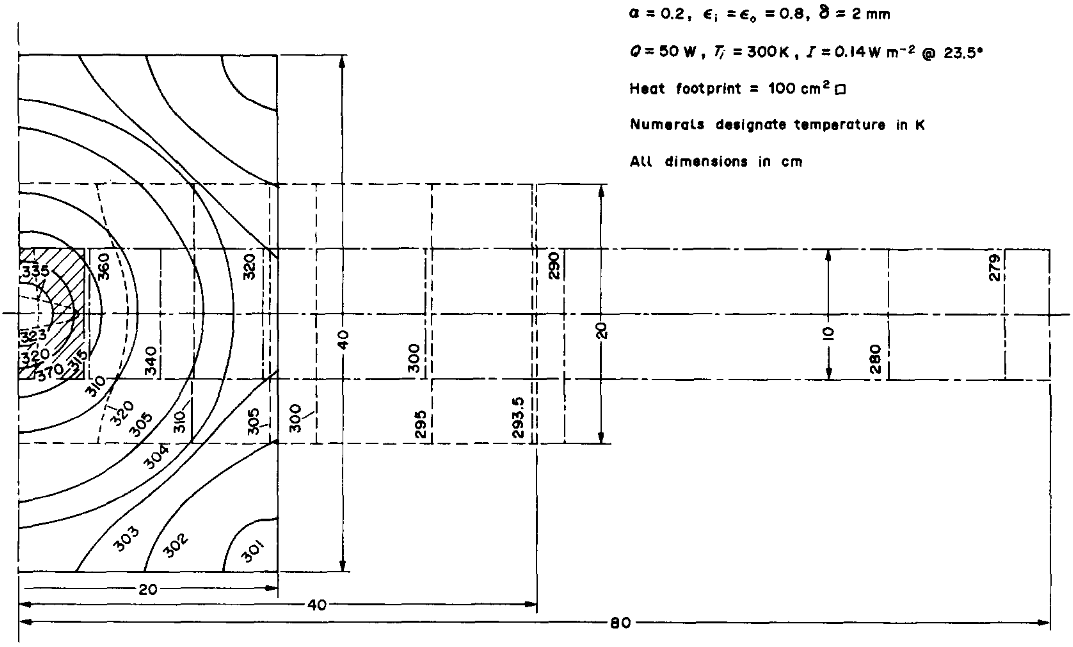

FIG. 6. Isotherms in rectangular plate (effects of aspect ratio).

sivity ratio W, and its effect on the temperature distribution is shown in Fig. 5. At $W = 0$, no heat is radiated to the ambient conditions. The mean temperature of the plate increases with a reduction in W.

Figure 6 depicts the isotherms in rectangular plates of equal area and different aspect ratios. A square heat footprint at the centre is considered in all the cases. The isotherms are circular except at the corners for a square plate. With the increase in the aspect ratio, the two-dimensional effect of temperature is predominant near the heat footprint area only. Also the maximum temperature at the origin increases with aspect ratio. The variation in the maximum temperature attained with change in footprint location is shown in Fig. 7. For a given heat load and radiating conditions a doubly symmetric location of the foot-

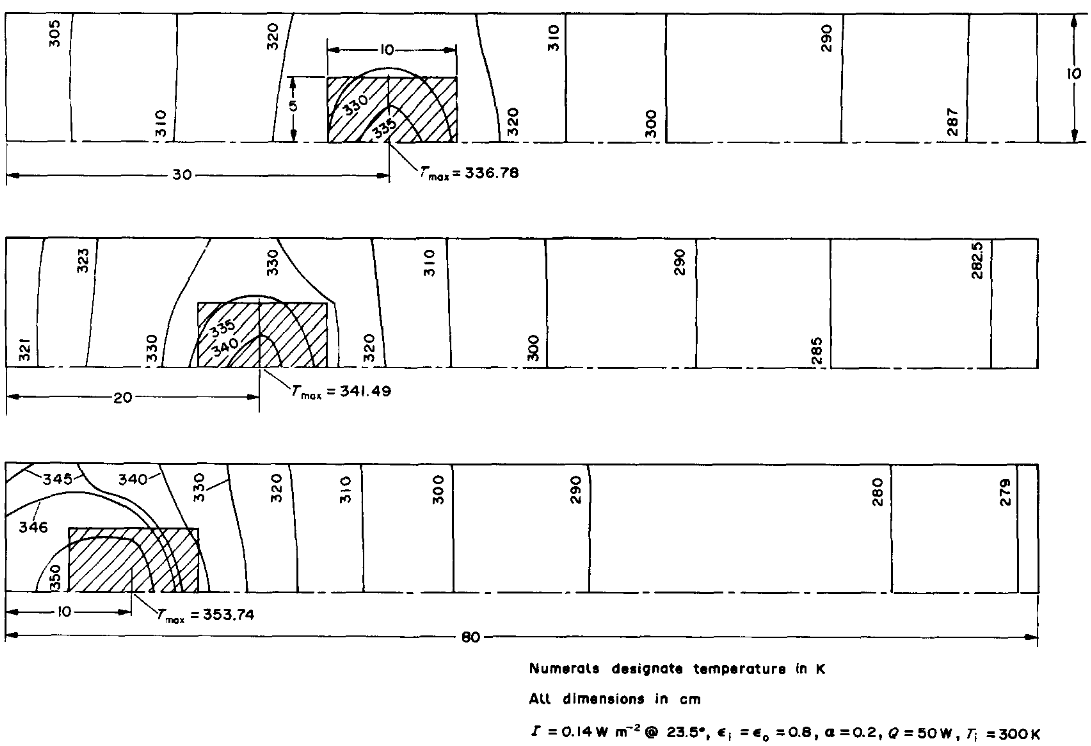

FIG. 7. Isotherms in rectangular plate (effects of heat footprint location).

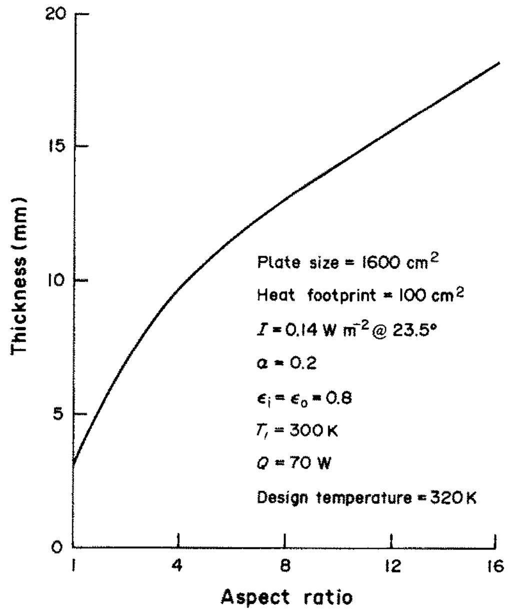

FIG. 8. Plate thickness vs aspect ratio.

4.1. *Design of radiating plates*

One of the primary objectives of thermal design is to maintain the temperature of the heat dissipating component at or below a specified value. If the shape of the plate and the heat footprint location is fixed, the above objective can be achieved by varying the thickness of the plate. This is generally done by a trial and error method. In the present investigation a one-dimensional search method is used for a quick estimation [16]. Figure 8 shows the thickness required for different aspect ratios. An increase in the maximum temperature attained with aspect ratio necessitates the increase in the thickness to contain the maximum temperature to a desired value. For values of aspect ratio greater than 6, the thickness required increases almost linearly.

Figure 9 shows two grid sizes used and the isotherms in an irregular hexagonal radiating plate with a centrally located square footprint area. The isotherms are circular at the centre and elongate to an elliptical shape in the body of the plate. A two-dimensional effect is seen throughout the plate area. The closer isotherms around the footprint area showing a larger temperature gradient are further apart near the edge showing a larger temperature range. The percentage difference in the temperature values at locations A, B and C on the plate for the two grid sizes of 21×21 and 11×11 are 0.27, 0.87 and 0.45, respectively.

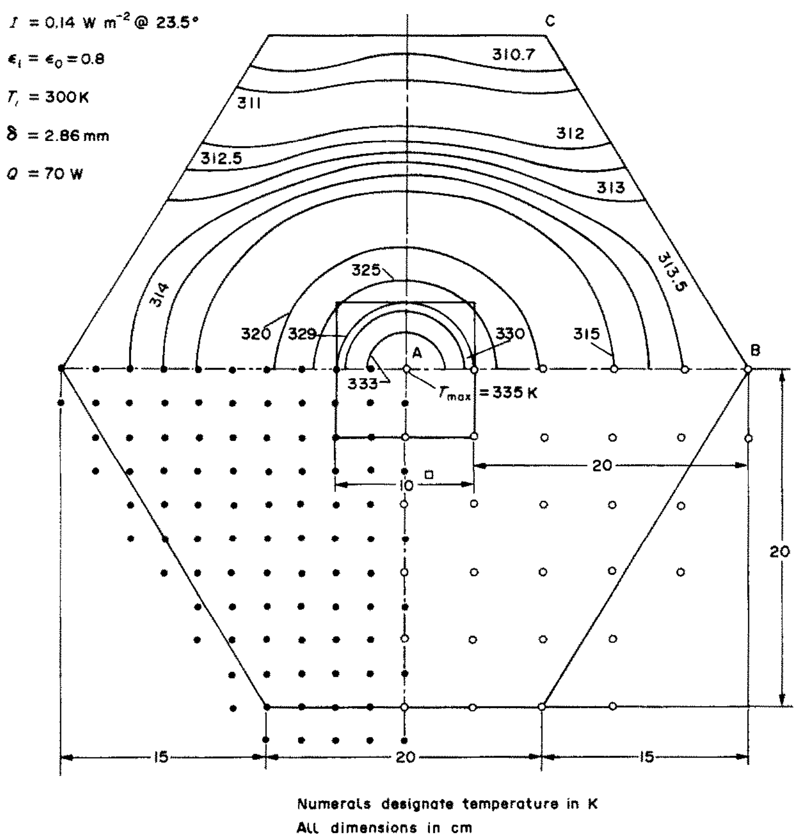

FIG. 9. Isotherms in an irregular hexagonal plate.

print and the plate will attain the lowest temperature at the origin. From the point of view of temperature constraints, as experienced in heat dissipation in electronic devices, the doubly symmetric configuration is best suited whenever possible.

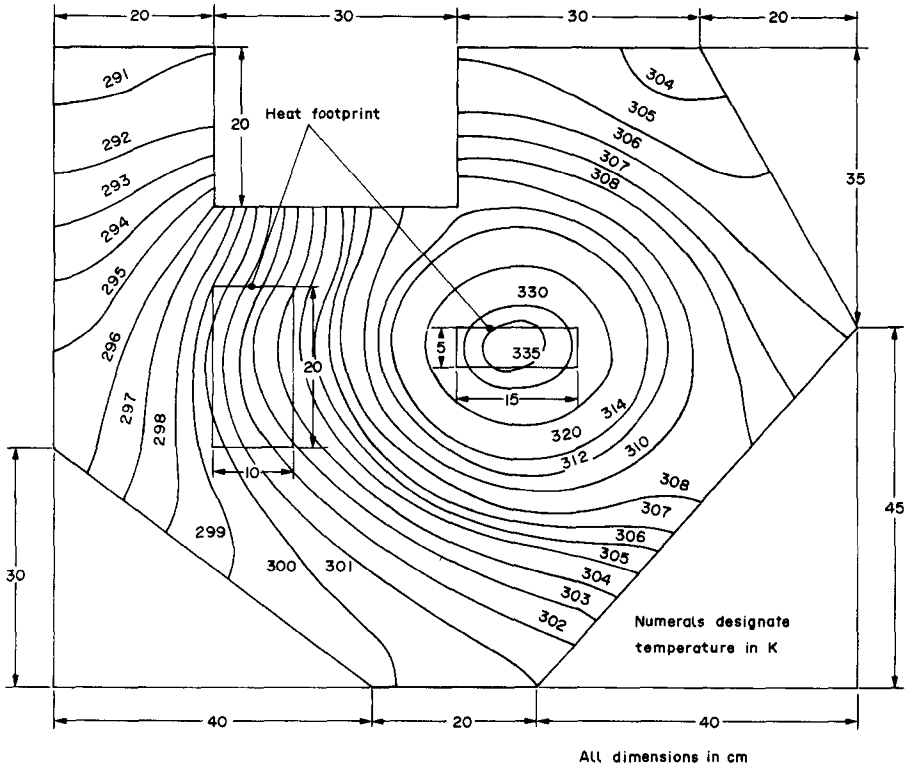

FIG. 10. Isotherms in an irregular plate.

In practice it may be required to use an irregular shaped plate (from the viewpoint of mass minimization) with more than one heat footprint. Figure 10 shows the isotherms in such a plate of size 5550 cm^2. The two heat footprint areas and load are 200, 75 cm^2 and 18, 150 W, respectively. The absorptivity, emissivity ratio of the external side facing the vacuum at 0 K is 0.2/0.8. The other side of the plate has an emissivity of 0.8 and radiates to an ambient condition at 300 K. As can be expected the isotherms which are closer near the high heat flux zone, spread out in the other region. Only one hot spot is observed due to the large difference in the applied heat flux values.

5. CONCLUDING REMARKS

The mathematical model presented in this paper leads to a better understanding of two-dimensional heat transfer in radiating plates. The effects of the non-dimensional parameters namely, the heat flux parameter, profile number, environmental parameter, emissivity ratio and aspect ratio upon the temperature distribution are studied.

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ANALYSE DU TRANSFERT THERMIQUE BIDIMENSIONNEL DES PLAQUES RADIANTES

Résumé—On utilise la procédure fortement implicite pour traiter la conduction de chaleur bidimensionnelle, non linéaire, elliptique dans des plaques radiantes. Un flux thermique uniforme est imposé à une face de la plaque qui dissipe la chaleur par rayonnement dans le vide à 0 K et dans une ambiance à la température T_i. Les résultats sont présentés à l'aide de quatre paramètres adimensionnels.

UNTERSUCHUNG DES ZWEIDIMENSIONALEN WÄRMETRANSPORTES IN EINER WÄRMEABSTRAHLENDEN PLATTE

Zusammenfassung—Ein implizites Verfahren wird zur Berechnung der nicht-linearen elliptischen zweidimensionalen Wärmeleitung in einer wärmeabstrahlenden Platte verwendet. Der Platte wird ein gleichmäßiger Wärmestrom aufgeprägt. Die Wärme wird auf der einen Seite der Platte in einen evakuierten Raum von 0 K abgestrahlt, auf der anderen Seite in eine Umgebung mit einer Temperatur T_i. Die Ergebnisse werden in Form von vier dimensionslosen Parametern dargestellt.

ДВУМЕРНЫЙ АНАЛИЗ ТЕПЛОПЕРЕНОСА ИЗЛУЧАЮЩИХ ПЛАСТИН

Аннотация—Для решения нелинейной эллиптической двумерной задачи теплопроводности для излучающих пластин используется неявный метод. Однородный тепловой поток подается на один конец пластины, которая рассеивает тепло излучением с одного конца в вакууме при $T = 0$ K и с другого конца в окружающую среду при температуре T_i. Результаты представлены с помощью четырех безразмерных параметров.